Model for Polarized and Unpolarized Parton Density Functions in the Nucleon

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Abstract

We present a physical model for polarized and unpolarized structure functions and parton density functions (PDFs) of the proton and the neutron. It reproduces the data on $F_2^p(x, Q^2)$ for $0.00001 < x < 1$ and $2.5 < Q^2 < 5000$ GeV^2, $F_2^p(x) - F_2^n(x)$, $F_2^n(x)/F_2^p(x)$, $xg(x)$, $\bar{d}(x) - \bar{u}(x)$, $d(x)/u(x)$, the Gottfried sum, the fractional momentum of charged partons and the polarized structure functions $g_1^{p,n}(x)$, at various Q^2. We present for the first time, proton and neutron PDFs which do not assume charge symmetry. Contrary to the common practice, we explain polarized and unpolarized data with a single model.

PACS numbers: 14.20.Dh, 13.60.Hb, 12.40.Ee

Keywords: Charge-symmetry violation, deep inelastic scattering, nucleon structure functions, parton density functions, phenomenological models of the nucleon, statistical models, finite-size effects

I. INTRODUCTION

Several new measurements of the polarized structure functions (SFs) of the proton and the neutron have been reported in the last few years [1]. In addition, recent experiments have significantly widened the kinematic range over which the unpolarized SFs and gluon density in the proton are known [2]. The New Muon Collaboration has obtained accurate, final results on the ratio of the deuteron and the proton (unpolarized) SFs, which can be used to extract the ratio and the difference of the proton and the neutron SFs [3]. There are new data also on the anti-down and anti-up quarks in the proton [4]. On the theoretical side, many parameterizations of unpolarized and polarized parton density functions (PDFs) exist [5]. Parameter values are determined from a global fit to the relevant high-energy data. As new data become available, these parameterizations are generally revised.

In this Letter, we adopt a very different approach. Our input PDFs are based on a different *ansatz* and have a few physically motivated free parameters. In principle, two of these parameters (a and b in (2)) could be evaluated theoretically, but in the absence of a full understanding of a quantum chromodynamic (QCD) bound state, namely the nucleon, we adopt the following pragmatic approach. We determine them by fitting data on the unpolarized structure function $F_2(x, Q^2)$ at *only one* value of Q^2. Here x is the Bjorken variable and Q^2 is the momentum scale. Thus we do not perform a global fit, with a large number ($\sim$ 15-20) of free parameters in the PDFs. We do not change the parameterization when we go from unpolarized to polarized PDFs. This is contrary to the common practice where they are parameterized separately. *Finally, in contrast to the commonly available PDF sets, our PDFs for the proton and the neutron are not based on the assumption of charge symmetry.* The issue of charge-symmetry violation (CSV) has acquired importance in recent months [6].

Once the parameters of the model are determined, it can successfully predict the rest of the unpolarized and polarized data, at various values of Q^2 and x. We have calculated: $F_2^p(x, Q^2)$ for $0.00001 < x < 1$ and $2.5 < Q^2 < 5000$ GeV^2, and the following observables at the various values of Q^2 for which data are available: $F_2^p(x) - F_2^n(x)$, $F_2^n(x)/F_2^p(x)$, $xg(x)$, $\bar{d}(x) - \bar{u}(x)$, $d(x)/u(x)$, the Gottfried sum S_G, the longitudinal momentum fraction carried by quarks and antiquarks, and finally the polarized structure functions $g_1^p(x)$ and $g_1^n(x)$. The agreement with experimental data is nearly as good as that obtained by the standard PDFs [5].

II. MODEL

There is enough evidence to seriously consider the *ansatz* that the input-scale parton densities in the nucleon may be quasi-statistical in nature [7,8]. It is also known that statistical mechanics is applicable to isolated quantum systems with finite numbers of particles if the residual two-body interaction is sufficiently strong and that the interaction-driven statistical equilibrium that emerges in such systems can be described in terms of the usual statistical quantities such as temperature [9]. We do not claim that the formalism in [9] is applicable *in toto* to the QCD bound state such as the nucleon. *We do, however, believe that* [7–9] *point towards the need for having an open mind about the above ansatz.* The success of a statistical model such as the present one, in reproducing and correlating a vast body of

polarized and unpolarized SF and PDF data provides a strong a posteriori justification for the above *ansatz*.

The parton number density dn^i/dx in the infinite-momentum frame (IMF) and the density dn/dE in the nucleon rest frame are related to each other by

$$\frac{dn^i}{dx} = \frac{M^2 x}{2} \int_{xM/2}^{M/2} \frac{dE}{E^2} \ \frac{dn}{dE}, \tag{1}$$

where the superscript i refers to the IMF, M is the nucleon mass and E is the parton energy in the nucleon rest frame [8]. *This is a general relation connecting the two frames; the only assumption made is that of massless partons.* This assumption is common in the formalism of deep inelastic scattering.

There is a standard procedure in statistical mechanics to introduce the effects of the finite size of an enclosure, in the expression for the density of states [10]. It allows us to write dn/dE for the nucleon as

$$dn/dE = g\ f(E)\ (VE^2/2\pi^2 + aR^2E + bR), \tag{2}$$

where g is the spin-color degeneracy factor, $f(E)$ is the usual Fermi or Bose distribution function $f(E) = \{\exp[(E-\mu)/T] \pm 1\}^{-1}$, V is the nucleon volume and R is the radius of a sphere with volume V. The three terms in (2) are the volume, surface and curvature terms, respectively; in the thermodynamic limit only the first survives. We determined the parameters a and b in (2) by fitting the structure function $F_2(x)$ data at a fixed momentum scale Q^2. Unlike a and b, the temperature (T) and chemical potential (μ) in the Fermi and Bose distributions are not fitted to a detailed shape of any data, but get determined due to number and momentum constraints on the PDFs (see below). Note also that a, b, T and μ are constants independent of x and Q^2.

We now depart from the procedure followed in [8] and present a more complete model for polarized as well as unpolarized PDFs and SFs of protons and neutrons. If $n_{\alpha(\bar{\alpha})\uparrow(\downarrow)}$ denotes the number of quarks (antiquarks) of flavor α and spin parallel (antiparallel) to the nucleon spin, then any model of PDFs in the proton has to satisfy the following seven constraints:

$$n_{u\uparrow} + n_{u\downarrow} - n_{\bar{u}\uparrow} - n_{\bar{u}\downarrow} = 2, \tag{3}$$

$$n_{d\uparrow} + n_{d\downarrow} - n_{\bar{d}\uparrow} - n_{\bar{d}\downarrow} = 1, \tag{4}$$

$$n_{s\uparrow} + n_{s\downarrow} - n_{\bar{s}\uparrow} - n_{\bar{s}\downarrow} = 0, \tag{5}$$

$$n_{u\uparrow} - n_{u\downarrow} + n_{\bar{u}\uparrow} - n_{\bar{u}\downarrow} = \Delta u, \tag{6}$$

$$n_{d\uparrow} - n_{d\downarrow} + n_{\bar{d}\uparrow} - n_{\bar{d}\downarrow} = \Delta d, \tag{7}$$

$$n_{s\uparrow} - n_{s\downarrow} + n_{\bar{s}\uparrow} - n_{\bar{s}\downarrow} = \Delta s, \tag{8}$$

$$\sum_{all\ partons} (\text{momentum} \ \ \text{fraction}) = 1, \tag{9}$$

and similarly for the neutron. The values of Δu, Δd and Δs in (6)-(8) have been measured by several groups. We use $\Delta u = 0.83 \pm 0.03$, $\Delta d = -0.43 \pm 0.03$, $\Delta s = -0.10 \pm 0.03$ for the proton and $\Delta u = -0.40 \pm 0.04$, $\Delta d = 0.86 \pm 0.04$, $\Delta s = -0.06 \pm 0.04$ for the neutron [1]. The summation in (9) runs over quarks, antiquarks and gluons. The numbers $n_{\alpha(\bar{\alpha})\uparrow(\downarrow)}$ in (3)-(8) are obtained from (1)-(2) by integrating the appropriate dn^i/dx over x, and the momentum

fractions in (9) are obtained similarly by integrating the appropriate xdn^i/dx over x. The double integrations are performed by first interchanging the order of the two integrations, and then doing the x integration analytically and the E integration numerically. It is necessary to distinguish between $\mu_{\alpha\uparrow}$ and $\mu_{\alpha\downarrow}$, to ensure that the statistical model is consistent with the constraints (6)-(8). We note that $\mu_{\bar{\alpha}\uparrow} = -\mu_{\alpha\downarrow}$ and $\mu_{\bar{\alpha}\downarrow} = -\mu_{\alpha\uparrow}$. Hence (3)-(9) represent 7 coupled nonlinear equations in 7 unknowns, namely $\mu_{u\uparrow}$, $\mu_{u\downarrow}$, $\mu_{d\uparrow}$, $\mu_{d\downarrow}$, $\mu_{s\uparrow}$, $\mu_{s\downarrow}$ and T. We can solve them numerically for a given choice of a and b. Once these 7 unknowns are determined, dn^i/dx are known, and polarized and unpolarized SFs can be evaluated using the standard relations between SFs and PDFs.

We have described above our model of the *input-scale* (Q_0^2) PDFs. Since data are available at various values of Q^2, it is necessary to be able to evolve these PDFs from Q_0^2 to any other Q^2. This involves evolving singlet, nonsinglet and gluon densities in the unpolarized and polarized cases. We have done this by solving the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations in the next-to-leading order, taking $Q_0^2 = M^2$, $N_f = 4$ and $\alpha_s(m_z^2) = 0.117$ [11].

III. RESULTS AND DISCUSSION

Our fits to the structure functions $F_2^p(x)$ and $F_2^n(x)$ at $Q^2 = 4$ GeV^2 are shown in Fig. 1a. We chose 4 GeV^2, because the NMC data [3] on $F_2^p - F_2^n$ are available only at this Q^2. The resultant values of the two parameters, namely a and b, are given in Table I. We shall comment on these values later. It is easy to show analytically using (3)-(8) that $\mu_{u\uparrow} > \mu_{d\downarrow} > \mu_{u\downarrow} > \mu_{d\uparrow} > \mu_{s\downarrow} = -\mu_{s\uparrow} > 0$, for the proton. Figure 1b shows our results for $F_2^p(x) - F_2^n(x)$ at $Q^2 = 4$ GeV^2 in comparison with the NMC data. The discrepancy between the calculated results and the data, near $x \simeq 10^{-2}$, is of the order of 0.005 which is negligible compared to $F_2^{p,n}$ in this region. The calculated Gottfried sum at $Q^2 = 4$ GeV^2, over the interval $0.004 < x < 0.8$, is 0.215 in agreement with the experimental number $0.221 \pm 0.019(\text{syst}) \pm 0.008(\text{stat})$ [14].

Once the parameters of the model are determined, it predicts the rest of the unpolarized and polarized data quite successfully without any further fitting. We now present these results. Figure 2 shows the calculated F_2^p in comparison with the data, in the range $0.00001 < x < 1$ and $2.5 < Q^2 < 5000$ GeV^2.

From SFs, we now turn to PDFs. The HERA and EMC data on $xg(x)$, the Fermilab data on $\bar{d}(x) - \bar{u}(x)$ and the CDHSW data on $d(x)/u(x)$ are compared, in Fig. 3, with our results and the results based on parameterizations in [5]. In particular, the $d(x)/u(x)$ ratio in the limit $x \to 1$ is found to be 0.22 in good agreement with the QCD prediction 0.2 [15]. The momentum fraction carried by quarks and antiquarks, at $Q^2 = 15$ GeV^2 is 0.58 which is consistent with the MRST result [5] and the experimental observation that the charged partons carry about half the proton momentum.

Finally, the results on the polarized structure functions $g_1^p(x)$ and $g_1^n(x)$ in Fig. 4, show that the model is able to predict the shapes of the polarized data, once the parameters are determined as explained in Sec. II. *The present model trivially satisfies the general positivity constraints on the polarized (δf) and unpolarized (f) PDFs*: $|\delta f(x, Q^2)| \leq f(x, Q^2)$.

We sum up the differences between the present model and that presented in [8]. (a) The earlier version did not distinguish between $\mu_\uparrow$ and $\mu_\downarrow$. It involved solving three simultaneous equations, namely the two number constraints and one momentum constraint, for the three

unknowns, namely T, μ_u and μ_d. It could not explain the polarized data. The present version splits up each chemical potential into two. It involves solving the seven simultaneous equations (3)-(9), and is able to explain and correlate the polarized and the unpolarized data by means of a single model. (b) We have presented here proton and neutron PDFs without assuming charge symmetry. Whereas the subject of charge symmetry violation in PDFs is being discussed extensively in the literature [6], we have presented PDFs which actually incorporate this feature. (c) The present model is in agreement with the observed difference and ratio of F_2^p and F_2^n, which was not the case earlier [8]. (d) In [8], the QCD evolution was performed to the leading order; here it is performed up to next-to-leading order.

Interestingly, the signs as well as magnitudes of a and b (Table I) turn out to be consistent with those in [10], suggesting a physical basis for our parameterization. Does the statistical model provide a physical basis to the commonly used PDF parameterizations [5]? To investigate, we have compared our PDFs with those in [5], at a common, low $Q^2 = 1.25$ GeV^2. Our xu_v and xd_v have the same shapes and similar magnitudes as those in [5]. Our xg is somewhat larger and $x\bar{u}$ and $x\bar{d}$ are somewhat smaller; however, their shapes are the same as in [5]. On this basis one would be inclined to answer the above question in the affirmative. Details will be published elsewhere.

In conclusion, we have shown that the ideas from statistical mechanics work even inside the nucleon. Trying to understand why they work, could deepen our understanding of hadron structure as well as statistical mechanics. Whereas all available PDFs assume charge symmetry, this paper presents, for the first time, a set of proton and neutron PDFs which does not make this simplifying assumption. The model is remarkably successful in reproducing a large body of polarized and unpolarized, PDF and SF data on the proton and the neutron. Thus it is potentially able to make *quantitative* predictions for the input PDFs. The scope and precision of the model can be extended systematically — e.g., by having a more elaborate treatment of the finite-size effects, by allowing finite mass of the charm quark, by considering nuclear corrections in the deuteron and higher-twist effects, etc.

We are very grateful to A. Deshpande, D. Fasching and E.-M. Kabuss for many useful communications.

Note added: Boros *et al.* [6] proposed a large CSV of the sea quarks in the nucleon, as an explanation of the discrepancy between neutrino (CCFR) and muon (NMC) nucleon structure function data at low x. This, however, has been criticized by Bodek *et al.* [6] who showed that the above proposal is ruled out by the published CDF W charge asymmetry measurements. This controversy does not affect the present work because the discrepancy, if any, between neutrino and muon data at low x is not used as an input anywhere in the model. The discussion of this issue in the recent literature was used only as one of the motivations for this model. The origin of CSV in the present model (i.e. the origin of the different values of a and b for the proton and the neutron) is in the tiny differences between $\Delta u, \Delta d, \Delta s$ values for the proton and the neutron (see the RHSs of Eqs. (6)-(8)), and also in the fact that we fit to F_2^p and F_2^n separately. These two points are independent of the above controversy.

TABLE I. Values of the parameters a and b. Temperature (T) and chemical potentials (μ) are in MeV. T and μ are constrained by (3)-(9) once a and b are specified.

	Proton	Neutron
(a, b)	$(-0.376, 0.504)$	$(-0.300, 0.504)$
T	62	59
$(\mu_{u\uparrow}, \mu_{u\downarrow})$	$(210, 86)$	$(40, 94)$
$(\mu_{d\uparrow}, \mu_{d\downarrow})$	$(42, 106)$	$(188, 76)$
$(\mu_{s\uparrow}, \mu_{s\downarrow})$	$(-7, 7)$	$(-4, 4)$

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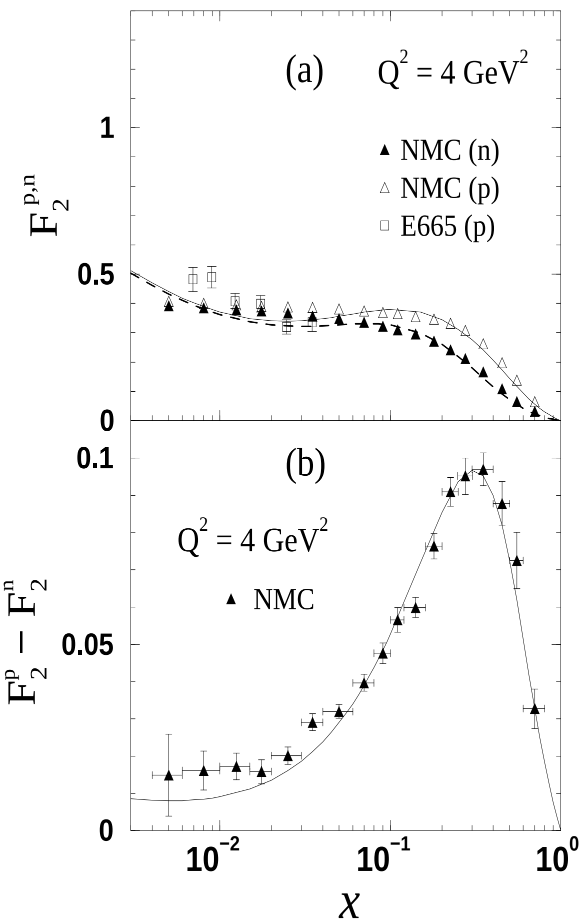

FIG. 1. (a) F_2^p and F_2^n *vs.* x. The solid and dashed curves are our fits to F_2^p and F_2^n data, respectively. The F_2^p data are from [12,13] and the F_2^n data are extracted from the accurate final results on $F_2^p - F_2^n$ at $Q^2 = 4$ GeV^2, supplied by the NMC collaboration [3]. NMC obtained these results by ignoring nuclear corrections in the deuteron. Since at low x, neutron data are not available, the proton data were used in that region, for the purpose of the fit. (b) $F_2^p - F_2^n$ *vs.* x. The curve represents our calculation and the data are from [3]. The small discrepancy between the curve and the data at low x, is discussed in the text.

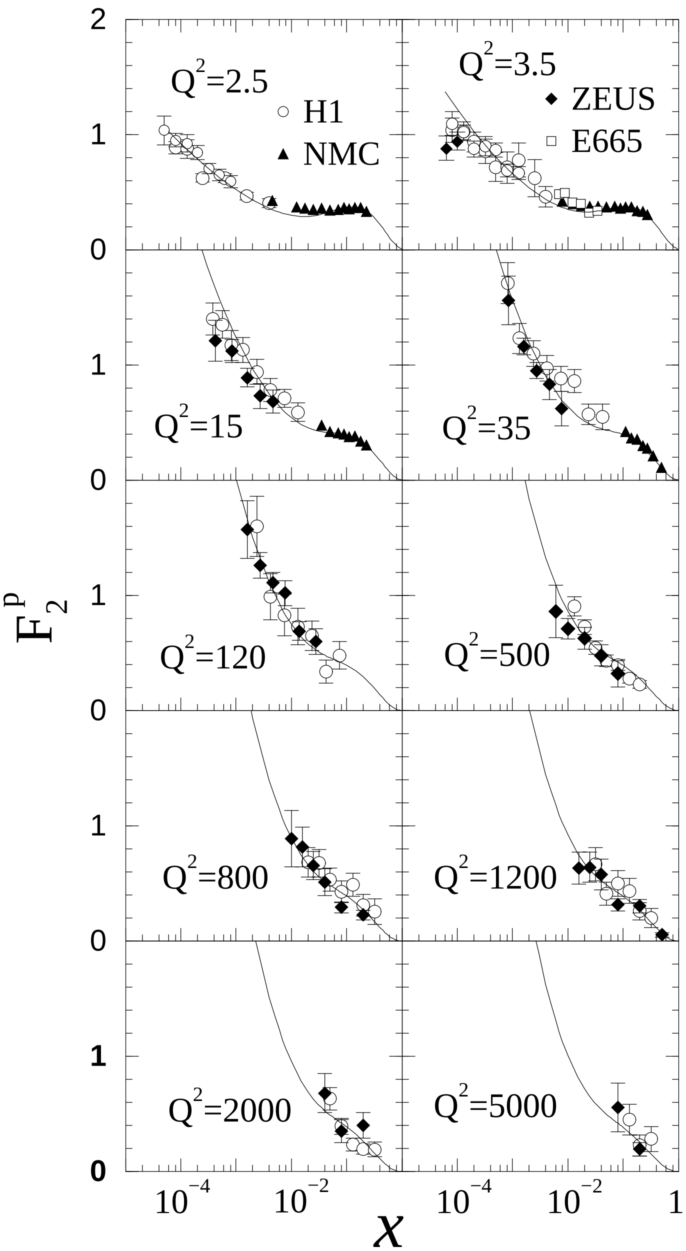

FIG. 2. Predictions of the model, for the unpolarized structure function F_2^p *vs.* x, over a wide range of Q^2 (GeV^2) and x, compared with experimental data [2,12,13].

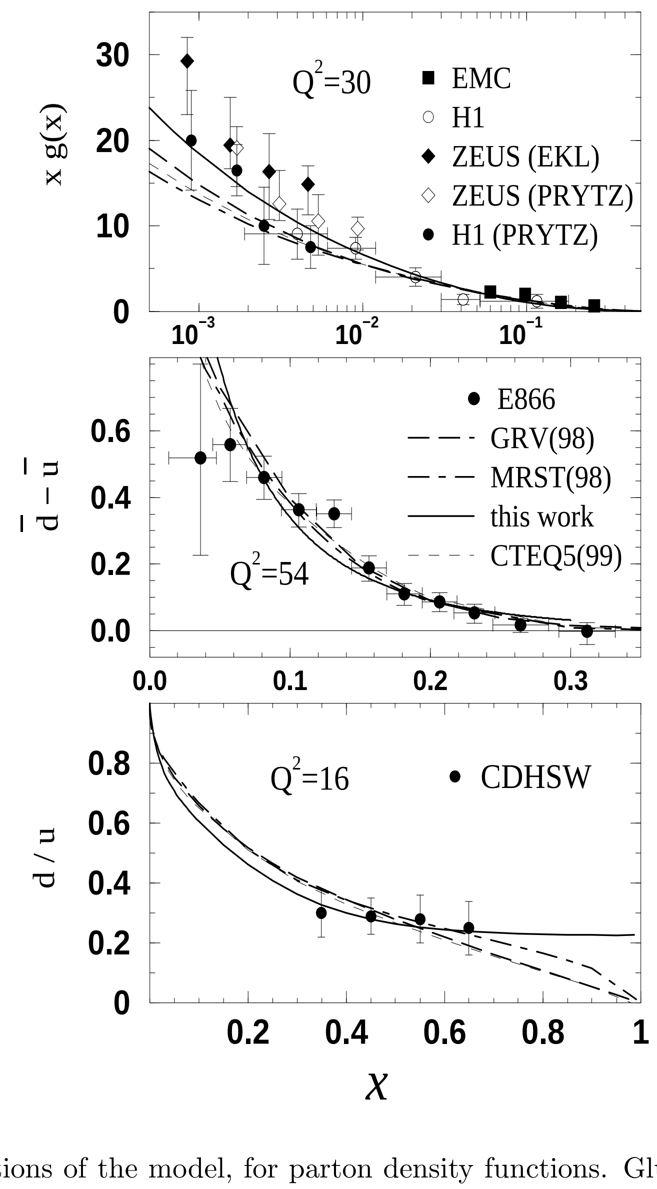

FIG. 3. Predictions of the model, for parton density functions. Gluon data from [16], $(\bar{d}-\bar{u})$ data from [4] and d/u data from [17]. In the top panel, data are at 20 and 30 GeV^2 while all the calculated results are at 30 GeV^2.

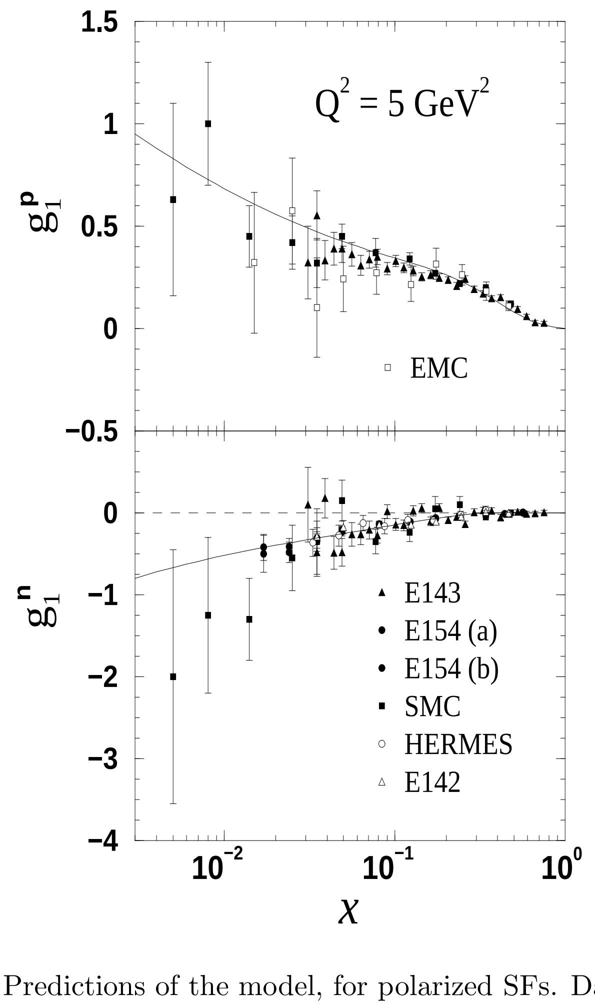

FIG. 4. Predictions of the model, for polarized SFs. Data are from [1].